MONITORING PROCESS STEPS USING VSSI EWMA CONTROL CHARTS

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1. Abstract

The article considers the variable process control scheme for two dependent process steps with incorrect adjustment. Incorrect adjustment of a process may result in shifts in process mean, ultimately affecting the quality of products. We construct variable sample size and variable sampling interval (VSSI) control charts to effectively monitor the quality variable produced by the first process step with incorrect adjustment and the quality variable produced by the second process step with incorrect adjustment. The performance of the proposed VSSI control charts is measured by the adjusted average time to signal (AATS) derived using a Markov chain approach. An example of an automobile braking system with incorrect adjustment shows the application and performance of the proposed VSSI control charts in detecting shifts in process mean. Furthermore, the performance of the specified VSSI control charts, the optimum VSSI and the fixed sample size and sampling interval (FSSI) control charts are compared by numerical analysis results. It has been found that the optimum VSSI control charts work better than both the specified VSSI control charts, and the specified VSSI control charts outperform the FSSI control charts. When quality engineers cannot specify the variable sample sizes and sampling intervals, the optimum VSSI control charts are suggested. Moreover, the impacts of misusing charts to monitoring the process mean and variance in the second step are also investigated.

2. Keywords: Control charts; dependent process steps; incorrect adjustment; Markov chain.

3. Introduction

Control charts are important tools in statistical quality control. They are used to effectively monitor and determine whether a process is in-control or out-of-control. A common problem in statistical process control is process adjusted unnecessarily (or overadjustment of a process) [6] due to incorrect use of control chart by the operator or since the only information about the state of the process is available through sampling. A process requires adjustment, when a control chart indicates that it is out of control. However, the process may be adjusted unnecessarily, when a false alarm occurs. Woodall [16] noted that the effect of overadjustment is a quite significant increase in variability of the quality characteristic. Collani and Saniga [3] proposed an adjustment model for the $\bar{X}$ control chart with a single special cause that considers the effects of process with incorrect adjustment from economic viewpoint. Their model determines the optimal design parameters of the $\bar{X}$ control chart which maximize the profitability of the process. Yang and Yang [18] addressed the economic adjustment model for $\bar{X}$ control chart with multiple special causes.

However, the above papers, even Shewhart $\bar{X}$ control charts, always monitor a process by taking equal sample sizes of a fixed sampling interval (FSI), they are usually slow in signaling small to moderate shifts in the process mean. Consequently, several alternatives have been developed to improve the performance of $\bar{X}$ control charts in recent years. One of the useful approaches to improve the detecting ability is to use a variable sampling interval (VSI) control chart instead of the traditional FSI. However, there is some indication that a process parameter may have changed, the next sampling interval should be shorter. On the other hand, if there is no indication of a parameter change, then the next sampling interval should be longer. There have been several alternatives developed to improve this problem in recent years (see Tagaras [14]).

The exponential weighted moving average (EWMA) control chart is a very effective alternative to the Shewhart control chart when small process shifts are of interest. The properties of EWMA charts with variable sampling intervals were studied by Shamma, Amin and Shamma [13], Amin and Letsinger [1], Saccucci, Amin, and Lucas [12], Reynolds [9], Reynolds [10, 11] and Capizzi and Masarotto [2]. Very little work has been done on variable sample size and sampling interval (VSSI) EWMA control charts for monitoring process mean. The properties of EWMA charts with variable sample size and sampling intervals were studied by Reynolds and Arnold [22], Park, Lee and Kim [23]. Tagaras [14] reviewed the literature on adaptive control charts. These papers show that most work on developing VSI or VSSI control charts has been done for the problem of monitoring process mean without considering the effects of incorrect adjustment of a process.

However, these articles assume that there is only a single process step, whereas many products are currently produced in several dependent process steps. Consequently, it is not appropriate to monitor these process steps by utilizing a control chart for each individual process step. Zhang [17] proposes the simple cause-selecting control chart to control the specific quality in the second process step by adjusting the effect of in-coming quality variable ($X$) on out-going quality variable ($Y$), since the in-coming quality variable on the first process step and the out-going quality variable on the second process step are dependent. The cause-selecting values ($e$) are $Y$ minus the effect of $X$, and the cause-selecting control chart is constructed accordingly. Wade and Woodall [15] review and analyze the cause-selecting control chart and examine the relationship between the cause-selecting control chart and the Hotelling $T^2$ control chart. In their opinion the cause-selecting control chart outperforms Hotelling $T^2$ control chart, since it is easy to
distinguish whether the second step is out-of-control. Yang [19] developed optimal control policy for dependent process steps with over-adjusted means. Yang and Yang [20] extended Yang’s approach to include over-adjusted variances. However, these papers with fixed control schemes. Therefore, it seems reasonable to develop variable control schemes to control dependent process steps. Yang and Yu [21] proposed VSI EWMA control charts to monitor dependent process steps. However, the properties of the VSI control charts used to control the process means on two dependent steps with incorrect adjustment have not yet been addressed. Therefore, to study the performance of the joint VSI EWMA control charts on two dependent process steps with incorrect adjustment is reasonable. In this paper, the joint VSI EWMA control charts are proposed to control the process means on dependent steps with incorrect adjustment. In next section, performance of the joint EWMA control charts is measured by the adjusted average time to signal (AATS), which is derived using a Markov chain approach. Finally, we illustrate the application of the proposed joint EWMA control charts using the example of automobile braking system with incorrect adjustment, and compare the performance among the joint optimum VSI EWMA control charts, joint specified VSI EWMA control charts and joint FSSI EWMA control charts. In case the variable sample sizes and sampling intervals cannot be specified by the engineers, the optimum VSI EWMA control charts are suggested.

4. Description of the Joint VSSI $\textit{EWMA}_x$ and $\textit{EWMA}_z$ Control Charts

Consider a process with two dependent process steps controlled by the joint VSSI $\textit{EWMA}_x$ and $\textit{EWMA}_z$ control charts. Let $X$ be the measurable in-coming quality variable on the first process step. Assume further that this process starts in a state of statistical control, that is, $X$ follows a normal distribution with the mean at its target value, $\mu_x$, and the standard deviation at its target value $\sigma_x$; let $Y$ be the measurable out-going quality characteristic of interest for the second process step, and follow a normal distribution conditional on $X$. Since the two process steps are dependent and the second process step is affected by the first process step, then following Wade and Woodall [16], the relationship between $X$ and $Y$ is generally expressed as

$$ Y_i | X_i = f ( X_i ) + \epsilon_i, i = 1, 2, 3, ..., m $$

where, it is assumed that $\epsilon_i \sim \text{NID}(0, \sigma_e^2)$. Let $Y$ instead of $Y_i | X_i$. If the function $f ( X_i )$ is known, the values of the standardized error term $\epsilon_i^* = Y_i - f ( X_i )/\sigma$ are called the cause-selecting values since they are the values of $Y_i$ adjusted for the effects of $X_i$. In practice, the true function $f ( X_i )$ is usually unknown and thus must be estimated using the $m$ observations obtained from the initial $m$ samples of size one, and thus the estimate for $f(X_i)$ will be $\hat{Y}_i$. The residuals, $\epsilon_i = Y_i - \hat{Y}_i$, are generated by the model used. Hence, $\epsilon_i \sim \text{NID}(0, \sigma_e^2)$. Consequently, the standardized residuals $\epsilon_i^* = \epsilon_i / \sigma_e$ are called the cause-selecting values.

However, in our study the chosen sample size is variable and the rational samples of size $n_q, (n_q > 1)$ are taken from the end of the two dependent process steps under variable sampling time intervals $(t_q > 0)$; the standardized sample means and variances, $Z_x$ and $Z_e$ are

$$ Z_x = \frac{\bar{X}_i - \mu_x}{\sigma_x / \sqrt{n_q}}, N(0, 1) \quad \text{and} \quad Z_e = \frac{\bar{e}_i}{\sigma_e / \sqrt{n_q}}, N(0, 1) $$

where

$$ \bar{X}_i = \frac{\sum_{j=1}^{n_q} X_j}{n_q} \quad \text{and} \quad \bar{e}_i = \frac{\sum_{j=1}^{n_q} e_{ij}}{n_q}, i = 1, 2, 3, ..., $$

Also assume that two special causes or process with incorrect adjustment may occur in either one of the two in-control process steps. The first step is only subject to the first special cause (SC1) or incorrect adjustment such that the mean of $X_y$ shifts from $\mu_y$ to $\mu_y + \delta_y \sigma_y$ ($\delta_y \neq 0$) but the variance has no changed, and the second step is only subject to the second special cause (SC2) or incorrect adjustment such that the mean of the specific quality ($e_{ij}$) shifts from 0 to $\delta_z$ ($\delta_z \neq 0$) but the variance has no changed. The out-of-control distribution of $X_y$ and/or $e_{ij}$ will be adjusted to in-control state, once at least one true signal is obtained from the proposed control charts. Let $T_{ni}$ be the time until the occurrence of the special cause i, and follow an exponential distribution of the form

$$ f(t_i) = \lambda_i \exp(-\lambda_i t_i), \quad t_i > 0, i = 1, 2. $$

where $1/\lambda_i$ is the mean time that the process step i remains in a state of statistical control.

To detect the small shifts in process means faster, the $\textit{EWMA}_x$ and $\textit{EWMA}_z$ control charts are constructed. Thus, The distributions of the statistics $\textit{EWMA}_x$ and $\textit{EWMA}_z$ should be derived. The statistics and distributions are as follows.
We divide the joint VSSI control charts into the following three regions.

The first region, within two warning limits, is called the central region. The second region, within warning limit and control limit, is called the control region.

Three VSSIs are adopted, \( 1 < n_1 < n_2 < n_3 < \infty \) and \( 0 < t_1 < t_2 < t_1 < \infty \). If the samples, \( EWMAX_1 \) and \( EWMAX_2 \), all fall within the central regions, then the next sample size should be small \( (n_1) \) and the next sampling interval should be long \( (t_1) \). If one sample falls within the central region but another falls within the warning region, then the next sample size should be median \( (n_2) \) and the next sampling interval should be median \( (t_1) \). If all samples fall within the warning regions, then the next sample size should be large \( (n_3) \) and the next sampling interval should be short \( (t_2) \).

Following Costa [5], the first sample size and sampling interval taken from the process when it is just starting is chosen randomly.

The position of the current sample in each control chart constructs the sampling interval of the next sample. We divide the joint VSSI \( EWMAX_1 \) and \( EWMAX_2 \) control charts into the following three regions.

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Following Costa [5], the first sample size and sampling interval taken from the process when it is just starting is chosen randomly. When the process is in control, all sample sizes and sampling intervals, including the first one, should have a probability of \( p_{a1} \) of being \((n_1,t_2)\), a probability of \( p_{a2} + p_{a3} \) of being \((n_2,t_2)\), and a probability of \( p_{a4} \) of being \((n_3,t_2)\), where \( \sum_{i=1}^{4} p_{ai} = 1 \), \( p_{a1} \), \( p_{a2} \), and \( p_{a4} \) are given by

\[
p_{a1} = \frac{\prod_{i=1}^{2} \text{Pr}(\text{EWMA}< k_{2-\lambda_1}, \text{EWMA}< k_{2-\lambda_1})}{\prod_{i=3}^{4} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})} = \frac{\prod_{i=1}^{2} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})}{\prod_{i=3}^{4} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})}
\]

\[
p_{a2} = \frac{\prod_{i=1}^{2} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})}{\prod_{i=3}^{4} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})} = \frac{\prod_{i=1}^{2} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})}{\prod_{i=3}^{4} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})}
\]

\[
p_{a4} = \frac{\prod_{i=1}^{2} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})}{\prod_{i=3}^{4} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})} = \frac{\prod_{i=1}^{2} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})}{\prod_{i=3}^{4} \text{Pr}(\text{EWMA}< k_{2-\lambda_1})}
\]
Based on the equation (5-1), the following equation can be formulated as

\[
p_{\text{in}} = P\left(-k_x \sqrt{\frac{2}{\lambda_A}} < \text{EWMA}_x < w_x \sqrt{\frac{2}{\lambda_A}} \right) \quad \text{or} \quad w_x \sqrt{\frac{2}{\lambda_A}} < \text{EWMA}_x < k_x \sqrt{\frac{2}{\lambda_A}} \quad \text{or} \quad k_x \sqrt{\frac{2}{\lambda_A}} < \text{EWMA}_x < \infty
\]

\[
= P\left(\text{EWMA}_x < w_x \sqrt{\frac{2}{\lambda_A}} \right) + P\left(\text{EWMA}_x > k_x \sqrt{\frac{2}{\lambda_A}} \right) = P_{\text{in}}
\]

Thus, \(P_{\text{in}}(\text{EWMA}_x < w_x \sqrt{\frac{2}{\lambda_A}}) = P(\text{EWMA}_x < k_x \sqrt{\frac{2}{\lambda_A}}) = 0\)

implying, \(w_x = k_x = k \) and \(A = A = \lambda_A\).

If \(w_x = w_x = 0\), \(n_1 = n_2 = n_3 = n_4\) and \(t_1 = t_2 = t_3 = t_4\), then the joint ASSI \(\text{EWMA}_X\) and \(\text{EWMA}_E\) charts reduce to the joint FSSI \(\text{EWMA}_X\) and \(\text{EWMA}_E\) charts with FSSI \(n_0\) and \(t_0\). If \(n_1 = n_2 = n_3 = n_4\) and \(t_1 < t_2 < t_3\), then the joint ASSI \(\text{EWMA}_X\) and \(\text{EWMA}_E\) charts reduce to the joint VSI \(\text{EWMA}_X\) and \(\text{EWMA}_E\) charts; if \(t_1 = t_2 = t_3 = t_0\) and \(n_1 < n_2 < n_3\), then the joint VSI \(\text{EWMA}_X\) and \(\text{EWMA}_E\) charts reduce to the joint variable sample size (VSS) \(\text{EWMA}_X\) and \(\text{EWMA}_E\) charts

5. Comparison of Control Charts

Sampling schemes should be compared under equal conditions; that is, VSSI and FSSI schemes should demand the same average sample size and sampling interval under the in-control period. That is, \(\text{ASSI} \text{EWMA}_X\) and \(\text{EWMA}_E\) charts when the process is in control. Thus,

\[
P_{\text{in}}(\text{EWMA}_x < w_x \sqrt{\frac{2}{\lambda_A}}) = P(\text{EWMA}_x < k_x \sqrt{\frac{2}{\lambda_A}}) = 0
\]

To facilitate the computation of the performance measures, \(w_x, k_x, w_x\) and \(k_x\) will be specified with the constraint that the probability of a sample falling in the central region is same for both the \(\text{EWMA}_X\) and \(\text{EWMA}_E\) charts when the process is in control.

Based on the equation (5-1), the following equation can be formulated as

\[
n_1 \cdot P(\text{EWMA}_x \in I_{-1} | \delta_1 = 0) + n_2 \cdot P(\text{EWMA}_x \in I_{-2} | \delta_1 = 0) + n_3 \cdot P(\text{EWMA}_x \in I_{-3} | \delta_1 = 0) + n_4 \cdot P(\text{EWMA}_x \in I_{-4} | \delta_1 = 0)
\]

Simplifying,

\[
4 \Phi(w)^3 \left[ e_1 - 2 e_2 + e_3 \right] + 4 \Phi(w)^2 \left[ e_2 - 2 e_3 + e_4 \right] \Phi(k) - 2 \Phi(k - 1) = 0
\]

where \(\Phi(\cdot)\) denotes the standard normal cumulative function.

The warning limit is derived as follows:

\[
w = \Phi \left[ -\frac{4B_t + 16B_t^2 - 16AC_t}{8A} \right]
\]

where

\[
\begin{align*}
\lambda_A &= n_1 - 2n_2 + n_3 \\
A &= n_0 + 2n_1 - 2n_2 - 2n_3 \\
B_t &= n_1 - 2n_4 \\
C_t &= \left[ n_0 (2\Phi(k) - 1) + n_2 + n_3 - 4n_4 \Phi(k) \right] - 4n_1 \Phi(k) - 4n_2 \Phi(k) - 4n_3 \Phi(k)
\end{align*}
\]

And from equation (3-1)

\[
\begin{align*}
t_1 \cdot P(\text{EWMA}_x \in I_{-1} | \delta_1 = 0) + t_2 \cdot P(\text{EWMA}_x \in I_{-2} | \delta_1 = 0) + t_3 \cdot P(\text{EWMA}_x \in I_{-3} | \delta_1 = 0) + t_4 \cdot P(\text{EWMA}_x \in I_{-4} | \delta_1 = 0) \\
+ t_1 \cdot P(\text{EWMA}_x \in I_{-2} | \delta_1 = 0) + t_2 \cdot P(\text{EWMA}_x \in I_{-3} | \delta_1 = 0) + t_3 \cdot P(\text{EWMA}_x \in I_{-4} | \delta_1 = 0) + t_4 \cdot P(\text{EWMA}_x \in I_{-4} | \delta_1 = 0) \\
+ t_1 \cdot P(\text{EWMA}_x \in I_{-3} | \delta_1 = 0) + t_2 \cdot P(\text{EWMA}_x \in I_{-4} | \delta_1 = 0) + t_3 \cdot P(\text{EWMA}_x \in I_{-4} | \delta_1 = 0) + t_4 \cdot P(\text{EWMA}_x \in I_{-4} | \delta_1 = 0)
\end{align*}
\]
Simplifying, 
\[ 4\Phi(w)\left(t_1 + 2t_2 + t_3\right) + 4\Phi(w)\left[-t_1 + 2\Phi(k) + t_3 - 2\Phi(k)\right] - t_1 - 4\Phi(k) + 4\Phi(k)w^2 = 0 \]

The warning limit can also be derived as follows:
\[ w = \Phi \left[ -\frac{4t_3 / k + \sqrt{16t_3^2 / k^2 - 16t_3k}}{8t_3} \right] \quad (5-3) \]

where 
- \( A_1 = t_2 - 2t_3 + t_4 \)
- \( B_1 = t_2 + 2\Phi(k) + t_3 - 2\Phi(k) \)
- \( C_1 = -\frac{4(2\Phi(k) - 1)t_3 - 4\Phi(k) - 4\Phi(k)w^2}{2} \)

However, to obtain \( w \) and let \( 0 < w < k \), the constraints \( 1 < t_1 < t_2 < t_3 \) and \( \theta < t_1 < t_2 < t_3 < \infty \) are required. Thus, the warning threshold \( w \) can be obtained by using equation (5-2) and choosing a specified combination of the three VSSs, \([n_1, n_2, n_3]\), and the FSS, \( n_0\).  

The determined warning threshold \( w \) and the specified combination of \([n_1, n_2, n_3, t_1, t_2]\) can be used to calculate the long sampling interval \( I_3 \) by using equation (5-4). That is,  
\[ t_i = \frac{\left(n_1 - n_j\right)^{t_i} - 1}{\sum_{j=1}^{32} n_j - n_i + 2} \left(\frac{2\Phi(w) - 1}{2\Phi(k) - 1}\right) \left(\frac{2\Phi(w) - 1}{2\Phi(k) - 1}\right) \left(n_j - n_i\right) \]  

In this paper, the VSSI scheme is compared with the FSSI scheme and one adaptive control scheme was considered to be better than another when it allows the joint VSSI EWMAt and EWMA\(\gamma\) charts to detect changes in the process means on step 1 and step 2 faster.

### 6. Performance Measurement

The speed with which a control chart detects process shifts measures the chart’s statistical efficiency. For a VSSI, the detection speed is measured by the average time from either mean shifting until either EWMAt and EWMA\(\gamma\) chart, or both signal, which is known as the AATS. That is, the AATS is the mean time that the process remains out of control.

Since \( TSC = \exp(-\gamma T) \quad \gamma > 0, i = 1, 2 \), the occurrence time, \( T_{0i} \), until the first special cause occurs is
\[ T_{0i} = \exp(TSC) \quad \text{where} \quad T_{0i} = \min(T_{SC1}, T_{SC2}) \]

Hence,  
\[ AATS = ATC - \frac{1}{T_{0i} + T_{0j}} \quad (6-1) \]

The average time of the cycle (ATC) is defined as the average time from the start of process until at least one true signal obtained from one or both of the proposed charts and the out-of-control process step 1 and/or step 2 are correctly adjusted. The Markov chain approach is allowed to compute the ATC due to the memory-less property of the exponential distribution. Thus, at each sampling, one of the 32 states is assigned based on whether the process step is in or out of control and the position of samples (see Table 4.1 for the 32 states of the process). The status of the process when the \((i+1)th\) sample is taken, and the position of the \(i\)th sample on the joint EWMAt and EWMA\(\gamma\) charts defines the transition states of the Markov chain. The joint VSSI EWMAt and EWMA\(\gamma\) charts produce a signal when at least one of the samples falls outside its control limits. If the current state is any of the States 1, 2, 4, 5, 10, 11, 12, 13, 17, 18, 20, 21, 24, 25, 27, 28 and 29 then the process steps are not adjusted and the current state may transit to any other state after sampling time interval \( I_m \) and take a sample of size \( n_m \), \( m=1,2,3 \). If the current state is State 3, it indicates one false signal comes from the second process step then the process is adjusted unnecessarily and State 3 instantly becomes any one of the States 10–13 with probability \( P_{0j}/j = 10 ~ 13 \) and, \( \sum_{j=10}^{13} P_{0j} = 1 \). Any one of the States 10–13 thus transits to any one of the States 10, 11, 12, 13 and 32 after a sampling time interval \( I_m \) and take a sample of size \( n_m \), with probability \( P_{i,j}/n_m, t_m \) and \( \sum_{j} P_{i,j}/n_m, t_m = 1 \), \( i = 10, \ldots, 13, j = 10,11,12,13,32, m = 1,2,3 \). State 6, 7, 8, 9, 14, 15, 16, 19, 22, 23, 26, 29, 30 and 31 are similar to State 3, since they indicate at least one false signal.

If the current state is any one of the States 1~31, then there is no true signal, hence States 1~31 are transient states. State 32 is reached when at least one true signal obtained from the out-of-control process step1 and/or step2. State 32 cannot transit to any other states, hence it is an absorbing state.

Denote \( P \) be the transition probability matrix, where \( P \) is a square matrix of order 32. Let \( P_{i,j}/n_m, t_m \) to be the transition probability from prior state \( i \) to the current state \( j \) with sample size \( n_m \) and sampling interval \( t_m \), where \( n_m \) and \( t_m \) are determined by the prior state \( i \), \( i = 1,2,3, \ldots, 32, j = 1,2, \ldots, 32, m = 1,2,3 \). The transition probability, for example, from state 1 to state 5 with sample size \( n_1 \) and sampling interval \( t_1 \) is calculated as
The calculation of all transition probabilities is shown in Appendix.

From the elementary properties of Markov chains (see, e.g., Cinlar [4]), the ATC is derived as follows:

\[ \text{ATC} = b^h(1-Q)^t + b(1-Q)^t A S \]

where \( b = \{ p_{00} p_{01} \ldots p_{031} \ldots 0 \} \) is the vector of starting probabilities for States 1, 2, \ldots, 31, where the first sampling interval has probability \( p_{0i} \) (see equation (2-6) for calculation) of being \((n, t_i)\) (or State 1 with probability \( p_{00} \)), the probability \( p_{00} + p_{0i} \) of being \((n, t_i)\) (or State 2 and State 4 with probability \( p_{02} \) and \( p_{04} \), respectively) and the probability \( p_{0i} \) of being \((n, t_i)\) (or State 5 probability \( p_{05} \)); \( I \) is the identity matrix of order 31; \( Q \) is the transition probability matrix where elements represent the transition probability, \( P_{ij}(n, t_i) \), from transient state \( i \), \( i=1, \ldots, 31 \), to transient state \( j \), \( j=1, \ldots, 31 \); \( M_i = (0, T, 0, 0, T, T, T, 0, 0, 0, T, T, T, T, 0, 0, T, T, T, T, 0, 0) \) is the vector of in-correct adjustment time for State 1 ~ State 31; \( t = (t_{11}, t_{12}, t_{13}, t_{14}, t_{15}, t_{21}, t_{22}, t_{23}, t_{24}, t_{25}, t_{31}, t_{32}, t_{33}, t_{34}, t_{35}, t_{41}, t_{42}, t_{43}, t_{44}, t_{45}, t_{51}, t_{52}, t_{53}, t_{54}, t_{55}) \) is the vector of the variable sampling intervals for state1-state 31, where \( t_{ji} \) is the average time of sampling interval for State 3, 6, 23, 30 and 31; \( t_{ji} \) is the average time of sampling interval for State 7, 8, 16, 24 and 29; \( t_{ji} \), \( t_{ji} \) and \( t_{ji} \) are shown in Appendix; \( A \) is the vector of transition probability, \( P_{ij}(n, t_i) \), form transient state \( i \), \( i=1, \ldots, 31 \), to absorbing state 32; \( t_{ji} \) is the time to adjust any process step correctly.

7. An Example

An example of process control for automobile braking system is presented, and the data of the process are measurements of roll weight and bake weight. Let variables \( X \) = roll weight and \( Y \) = bake weight be measured from the end of the second process step. The bake weight produced in the second step is influenced by the roll weight produced in the first step. Two machines are used in the process steps. One machine could only fail in the first process step and shift the mean of \( X \) distribution, and another machine could only fail in the second process step, and shift the mean of \( Y \) distribution. Presently, the joint FSSI control charts are used to monitor the shifts of mean on the two process steps every hour (10 samples indicate that the data follows bivariate normal distribution. The relationship of quality variables \( X \) and \( Y \) is expressed by a linear regression model. The fitted model is

\[ Y = 30.3 + 0.812X \]  

(7-1)

Thus, the residuals or specific quality (\( e \)) are obtained by \( y - 30.3X \). The estimated means and standard deviations of variables \( X \) and \( e \) are (\( \hat{\mu}_X = 210.5 \), \( \hat{\sigma}_X = 1.435 \)) and (\( \hat{\mu}_e = 0 \), \( \hat{\sigma}_e = 0.817 \)), respectively. That is, when both process steps are in-control, \( X \sim N(210.5, 1.435^2) \) and \( e \sim N(0,0.817^2) \) From historical data, the estimated failure frequency is 0.03 time per hour (or \( \gamma_1 = 0.03 \)) for machine 1 and 0.06 time per hour (or \( \gamma_2 = 0.06 \)) for machine 2. The failure machine 1 and 2 are independent and only influence the means of \( X \) and \( Y \), but the standard deviations are unaffected. The failure machine 1 would shift the mean of \( X \) to \( \hat{\mu}_X + \delta \hat{\sigma}_X \) where \( \delta = 0.75 \). The failure machine 2 would shift the mean of \( e \) to \( \hat{\delta} \hat{\sigma}_e \) where \( \hat{\delta} = 0.5 \). Hence, for out-of-control process step 1, \( X \sim N(210.5 + 0.75 \times 1.435, 1.435^2) \); for out-of-control step 2, \( e \sim N(0.5 \times 0.817, 0.817^2) \).

The FSSI \( EWMA_X \) and \( EWMA_e \) charts have control limits placed at \( \pm 2.492 \) and \( \lambda = 0.05 \) with average run length 370 when the average incorrect adjustment time of any process step is zero (or \( T_j = 0 \) h) and the average correct adjustment time of any process step is zero (or \( T_j = 0 \) h) (see Montgomery (2005)), respectively. In the study, \( T_j = 2 \) h when at least one false signal occurs, and \( T_j = 4 \) h when at least one true signal occurs. The AATS of the FSSI \( EWMA_X \) and \( EWMA_e \) charts is 2.6087h. The slowness with which the FSSI \( EWMA_X \) and \( EWMA_e \) control charts detect shifts in the process (\( \delta = 0.75, \hat{\delta} = 0.5 \)) has led the quality manager to propose building the \( EWMA_X \) and \( EWMA_e \) control charts with VSSI. The construction and application of the proposed VSSI \( EWMA_X \) and \( EWMA_e \) control charts is illustrated. The following are the guidelines for using the proposed charts:

Step 1. Let the factor of control limits, \( k = 2.492 \) and \( \lambda = 0.05 \), to maintain the in-control average run length is 370 for each \( EWMA_X \) and \( EWMA_e \) control chart when \( T_j = 0 \) h and \( T_j = 0 \) h.
Step 2. Since \(1 < n_1 < n_2 < n_3 < \infty\) and \(0 < t_2 < t_3 < \infty\) are required, and for performance of process control engineers adopt the combination \(n_1 = 2, n_2 = 5, n_3 = 12, t_1 = 0.001h, t_2 = 0.1h\) and \(t_3 = 1.0 h\).

Step 3. Letting \(t_1 = 0.001h, t_2 = 0.1h, t_3 = 1.0h, k = 2.4922\) and \(\lambda = 0.05\) in the equation (5-2) leads to \(\nu = 0.8357\). Consequently, the proposed VSSI \(EWMAx \) and \(EWMAL\) control charts are constructed by using the in-control 31 samples.

\[
UCL_{ewma} = 0.3990 \\
UWL_{ewma} = 0.1338 \\
CL_{ewma} = 0 \\
LWL_{ewma} = -0.1338 \\
LCL_{ewma} = -0.3990
\]

With the determined design parameters, the VSSI \(EWMAX\) and \(EWMAL\) control charts can be used for controlling the two dependent process steps for producing automobile braking. According to the VSSI scheme, if both samples \((EWMAX\) and \(EWMAL\)) fall within warning limits then the small sample size \((n_1 = 2)\) and the long sampling interval \((t_3 = 1.0 h)\) are taken. If one of the samples falls within the warning limits but the other falls between warning and control limits then a median sample size \((n_2 = 5)\) and a median sampling interval \((t_2 = 0.1 h)\) are taken. If both samples fall between warning and control limits, then the large sample size \((n_3 = 12)\) and short sampling interval \((t_1 = 0.001 h)\) are taken. If at least one sample falls outside the control limits of any proposed control chart, then the process steps are stopped and adjusted. The AATS is used to measure the performance of the proposed VSSI control charts. The proposed Markov chain approach is used to obtain the ATC and calculate the AATS. The AATS is 0.0269h. The VSSI scheme improves the sensitivity of the joint FSSI control charts. Hence, the specified VSSI control charts are constructed by using the in-control 31 samples.

\[
X_{ewma} = \frac{1}{n} \sum_{i=1}^{n} X_i \\
eWMA = X_{ewma} + (1-\lambda)eWMA
\]

An example using the constructed \(EWMAX\) and \(EWMAL\) charts with the VSSIs to monitor the dependent steps is introduced now. When the process starts, a random procedure decides the first sampling interval \(t_2 = 0.1h\) with sample of size five, and the first sample with the values of \(Z_X = 1.7442\) and \(Z_y = 0.4489\). Thus, their \(EWMAX\) and \(EWMAL\) values are calculated as follows.

\[
EWMAX_{\sigma_{2,1}} = Z_{x_2} + (1-\lambda)eWMA = 0.05 \cdot 1.7442 + 0.95 \cdot 0 = 0.0871 \\
EWMAX_{\sigma_{2,0}} = Z_{x_2} + (1-\lambda)eWMA = 0.05 \cdot (-0.4489) + 0.95 \cdot 0 = -0.0225
\]

Since both samples fall within the warning limits, the second sample will be observed adopting a sample of size two after the long sampling interval \(t_2 = 1.0 h\). The second sample is \((x = 211, y = 203)\). Since \(Z_X = 0.3484\) and \(Z_y = 1.5399\), so \(EWMAX\) and \(EWMAL\) values are calculated as follows.

\[
EWMAX_{\sigma_{3,1}} = Z_{x_3} + (1-\lambda)eWMA = 0.05 \cdot 0.3484 + 0.95 \cdot 0.0871 = 0.1002 \\
EWMAX_{\sigma_{3,0}} = Z_{x_3} + (1-\lambda)eWMA = 0.05 \cdot 1.5399 + 0.95 \cdot (-0.0255) = 0.0557
\]

Both samples fall within the warning limits, the third sample will be observed adopting a sample of size two after the long sampling interval \(t_3 = 1.0 h\). We find that all \(EWMAX\) values fall within the VSSI \(EWMAX\) chart, but the 35th \(EWMAL\) value falls outside the VSSI \(EWMAL\) chart (Fig. 7.1-7.2). It indicates that the process step 1 is in control, but the process step 2 is out of control on the 35th sample. Hence, the process step 2 is stopped and machine 2 is adjusted.

8. PERFORMANCE COMPARISON BETWEEN ASSI AND FSSI SCHEMES

Table 8.1 provides the AATS of the specified VSSI and FSSI schemes, which are obtained under various combinations of parameters based on orthogonal array \(L_{27}(3^{12})\) table, \(\gamma_1 = 0.03 \sim 0.05, \gamma_2 = 0.05 \sim 0.10, \delta = 0.01 \sim 1.0, \delta_l = 0.01 \sim 1.0, t_3 = 1.0 h, t_2 = 1.0 \sim 2.0, t_1 = 0.1 \sim 0.9, t_0 = 0.001 \sim 0.09, t_4 = 2 \sim 4, t = 4 \sim 6\) and \(n_0 = 5\).

Comparing the AATS between the FSSI and the specified VSSI \(EWMAX\) and \(EWMAL\) control charts, it can be seen that the performance of the specified VSSI \(EWMAX\) and \(EWMAL\) control charts is better for detecting small in process means \((0.01 \leq \delta \leq 1.0, 0.01 \leq \delta_l \leq 1.0)\). The specified VSSI \(EWMAX\) and \(EWMAL\) control charts save detection time from 10.33% to 98.97% compared to the FSSI \(EWMAX\) and \(EWMAL\) control charts. Hence, the specified VSSI \(EWMAX\) and \(EWMAL\) control charts outperform the FSSI \(EWMAX\) and \(EWMAL\) control charts.

Sometimes, quality engineers cannot specify the VSSIs. The optimal VSSIs of the proposed charts are thus suggested. The optimal VSSI of the proposed charts are determined using Quasi-Newton optimization technique (Fortran IMSL BCONF subroutine) to minimum AATS under the same constraints and parameters as described before. We find that the optimum VSSI \(EWMAX\) and
EWMA control charts save detection time from 28.17% to 99.92% compared to the FSSI EWMA control charts, and the optimum VSSI EWMA and EWMA control charts also work better than the EWMA control charts with specific variable sample sizes and sampling intervals. Furthermore, it should be noted that the optimum variable sample sizes \((n_1 = 2, n_2 = 5, n_3 = 9)\) and sampling intervals \((t_i^* = 0.001, t_i^* = 0.001, t_i^* = 1.001)\) are almost robust to the 27 combinations of parameters. It indicates that engineers should adopt the three variable sample sizes, \((n_1 = 2, n_2 = 5, n_3 = 9)\), but two variable sampling intervals \((t_i = 0.001 = t_i, t_i = 1.001)\) for obtaining the better performance of the VSSI EWMA and EWMA control charts.

9. Conclusions

The proposed VSSI scheme controlling two dependent process steps with incorrect adjustment substantially improves the performance of the FSSI scheme by increasing the speed with which small shifts in the means of process steps are detected. We have found that the optimum VSSI EWMA and EWMA control charts always work better (in the cases examined) than the specified VSSI and the FSSI EWMA and EWMA control charts for small \(\delta_1\) and \(\delta_2\) values. The optimum VSSI scheme controlling two dependent process steps with incorrect adjustment is thus suggested when quality engineers cannot specify the VSSIs.

This paper considered two dependent process steps with incorrect adjustment. However, a study of the variable parameters (V) EWMA and EWMA control charts under two dependent process steps with incorrect adjustment or economic V EWMA and EWMA control charts is an interesting topic for future research. Other important extensions of the proposed model can also be developed. It is straightforward to extend the proposed model to study V control charts or other control charts, such as attribute charts, CUSUM-charts or multivariate charts.

10. Tables and Figures

Table 8.1 AATS of the optimum VSSI and FSSI charts under various combinations of parameters

<table>
<thead>
<tr>
<th>Combination of parameters</th>
<th>VSSI</th>
<th>FSSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 0.20 0.02 0.04 0.01 0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2. 0.20 0.02 0.04 0.01 0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3. 0.20 0.02 0.04 0.01 0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4. 0.20 0.02 0.04 0.01 0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5. 0.20 0.02 0.04 0.01 0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

11. References